Distributed Target Optimization in SimNIBS

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1 Introduction

Using Trancranial Electric Stimulation (TES) methods for targeting cortical networks obtained from resting state fMRI (rs-fMRI) analyses has shown promising results [1]. However, as such networks feature spatially distributed target fields, it can be challenging to manually set-up simulations that approach the desired field distribution in the bran.

Approaches to automatically obtain optimal electrode montages in such multifocal setting have been previously proposed [2]. Such methods might at a first glance look very different from approaches for TES targeting where we try to constrain the electric field to few spatially defined targets [3]. In this report we will first show that both problems are tightly related, and can therefore be solved using many of the same methods. We will then go on into detail more detail as to how distributed target optimization was implemented in SimNIBS and show a few exemplary results.

2 Methods

2.1 Definitions

Consider we have a $N_{\text{roi}} \times 1$ vector $\boldsymbol{t}$ representing a target field distribution in a region of interest. Such map could be, for example T-values obtained from rs-fMRI functional network analysis. Following the approach proposed in [2] we will use $\boldsymbol{t}$ to define a diagonal $N_{\text{roi}} \times N_{\text{roi}}$ weight matrix $\boldsymbol{W}$

$$W_{ii} = \begin{cases} |t_i|, & \text{if} \quad |t_i| > t_{\text{min}} \\ t_{min}, & \text{else} \end{cases} \tag{1}$$

and a $N_{\text{roi}} \times 1$ vector $\boldsymbol{y}$

$$y_i = \begin{cases} E_0 t_i, & \text{if} \quad |t_i| > t_{\min} \\ 0, & \text{else} \end{cases} \tag{2}$$

where $t_{\min} \geq 0$ represents a minimum t-value and E_0 a target electric field, both selected by the user.

Following [2], we want to reduce the Error Relative to No Intervention (ERNI) metric, defined by

$$\Delta = \frac{\sum_{i=1}^{N_{\text{roi}}} (y_i - W_{ii} e_i)^2 - y_i^2}{\frac{1}{N_{\text{roi}}} \sum_{i=1}^{N_{\text{roi}}} W_{ii}^2}, \tag{3}$$

where $\boldsymbol{e}$ is the $N_{\text{roi}} \times 1$ vector with electric field normal components in the region of interest. Using a leadfield [3], we can write $\boldsymbol{e}$ as a matrix-vector multiplication

$$\boldsymbol{e} = \boldsymbol{A}\boldsymbol{x}. \tag{4}$$

Where $\boldsymbol{A}$ is an $N_{\text{roi}} \times N_{\text{elec}}$ matrix with the average-referenced normal electric field components. This matrix is constructed by assembling the fields obtained with each electrode. $\boldsymbol{x}$ is an $N_{\text{elec}} \times 1$ vector with electric current values at each electrodes. Substituting Equation 4 into Equation 3 and using a matrix notation, we obtain

$$\Delta = \frac{(\boldsymbol{y} - \boldsymbol{W}\boldsymbol{A}\boldsymbol{x})^2 - \boldsymbol{y}^2}{\frac{1}{N_{\text{roi}}} \text{trace}(\boldsymbol{W}^2)}. \tag{5}$$

Removing terms that do not depend on the optimization variable $\boldsymbol{x}$, we obtain

$$\Delta \propto (\boldsymbol{y} - \boldsymbol{W}\boldsymbol{A}\boldsymbol{x})^2 \tag{6}$$

$$\propto -2\boldsymbol{y}^\top \boldsymbol{W}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{x}^\top \boldsymbol{A}^\top \boldsymbol{W}^\top \boldsymbol{W}\boldsymbol{A}\boldsymbol{x} \tag{7}$$

$$\propto \boldsymbol{l}^\top \boldsymbol{x} + \boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x} \tag{8}$$

ERNI is therefore a quadratic function with respect to $\boldsymbol{x}$ where

$$\boldsymbol{l} = -2\boldsymbol{A}^\top \boldsymbol{W}^\top \boldsymbol{y}, \tag{9}$$

$$\boldsymbol{Q} = \boldsymbol{A}^\top \boldsymbol{W}^\top \boldsymbol{W}\boldsymbol{A}. \tag{10}$$

2.2 Optimization Problems

Introducing the constraints from [3], we have the optimization problem of minimizing ERNI

Optimization Problem 1

$$\begin{aligned} \text{minimize} \quad & \Delta && \text{(P1a)} \\ \text{such that} \quad & \mathbf{1}^\top \boldsymbol{x} = 0 && \text{(P1b)} \\ & \|\boldsymbol{x}\|_1 \leq 2\mathrm{I_{tot}} && \text{(P1c)} \\ & |x_i| \leq \mathrm{I_{ind}}, \quad i = 1, \dots, n && \text{(P1d)} \\ & \|\boldsymbol{x}\|_0 \leq N && \text{(P1e)} \end{aligned}$$

Where Constraint P1b enforces Kirchhoff's Current Law, Constraint P1c limits the total amount of currents injected to some value I_{tot}, Constraint P2d limits the currents injected through each electrode to some value I_{ind} and Constraint P1e limits the number of electrodes used to N. For more information about this formulation, please see [3].

Using the development in Equations 6-10, we can write an equivalent optimization problem

Optimization Problem 2

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{l}^\top \boldsymbol{x} + \boldsymbol{x}^\top \boldsymbol{Q} \boldsymbol{x} && \text{(P2a)} \\ \text{such that} \quad & \mathbf{1}^\top \boldsymbol{x} = 0 && \text{(P2b)} \\ & \|\boldsymbol{x}\|_1 \leq 2\mathrm{I_{tot}} && \text{(P2c)} \\ & |x_i| \leq \mathrm{I_{ind}}, \quad i = 1, \dots, n && \text{(P2d)} \\ & \|\boldsymbol{x}\|_0 \leq N && \text{(P2e)} \end{aligned}$$

Where $\boldsymbol{l}$ and $\boldsymbol{Q}$ are defined in Equations 9 and 10, respectively.

Problem 2 is of the same class as Problems 4 and 10 in [3] and can therefore be solved using the same methods, including the Branch-and-Bound algorithm to enforce the limit to the number of electrodes (Constraint P2e).

References

[1] D. B. Fischer, P. J. Fried, G. Ruffini, O. Ripolles, R. Salvador, J. Banus, W. Ketchabaw, E. Santarnecchi, A. Pascual-Leone, and M. D. Fox, "Multifocal tdcs targeting the resting state motor network increases cortical excitability beyond traditional tdcs targeting unilateral motor cortex," *Neuroimage*, vol. 157, pp. 34–44, 2017.

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